UNIVERSIDAD NACIONAL DE INGENIERÍA

Facultad de Ingeniería Mecánica

MA 101 Matemáticas Aplicadas

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B *Apollo* Reentry

Courtesy of Alar Toomre, Massachusetts Institute of Technology

Each time the Apollo astronauts returned from the moon circa 1970, they took great care to reenter Earth's atmosphere along a path that was only a small angle α from the horizontal. (See Figure 4.37.) This was necessary in order to avoid intolerably large "g" forces during their reentry.

To appreciate their grounds for concern, consider the idealized problem

$$\frac{d^2s}{dt^2} = -Ke^{s/H}\left(\frac{ds}{dt}\right)^2 ,$$

where K and H are constants and distance s is measured downrange from some reference point on the trajectory, as shown in the figure. This approximate equation pretends that the only force on the capsule during reentry is air drag. For a bluff body such as the *Apollo,* drag is proportional to the square of the speed and to the local atmospheric density, which falls off exponentially with height. Intuitively, one might expect that the deceleration predicted by this model would depend heavily on the constant K (which takes into account the vehicle's mass, area, etc.); but, remarkably, for capsules entering the atmosphere (at "$s = -\infty$") with a common speed V_0, the *maximum* deceleration turns out to be independent of K.

(a) Verify this last assertion by demonstrating that this maximum deceleration is just $V_0^2/(2eH)$. [*Hint:* The independent variable t does not appear in the differential equation, so it is helpful to make the substitution $v = ds/dt$; see Project A, part (b).]

(b) Also verify that any such spacecraft at the instant when it is decelerating most fiercely will be traveling exactly with speed $V_0/\sqrt{e}$, having by then lost almost 40% of its original velocity.

(c) Using the plausible data $V_0 = 11$ km/sec and $H = 10/(\sin\alpha)$ km, estimate how small α had to be chosen so as to inconvenience the returning travelers with no more than 10 g's.

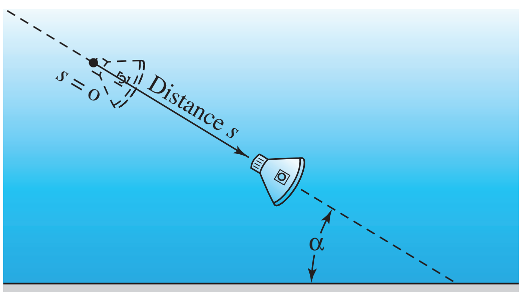

Figure 4.37 Reentry path

Ayuda: Usar $dv/ds = (dv/dt)(dt/ds) = (d^2s/dt^2)(1/v)$.